

# Neutrino Decay of Positronium in the Standard Model and Beyond

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## Abstract

Whether in the Standard Model or beyond it, neutrinos contribute to the invisible decay mode of orthopositronium but practically not at all to that of parapositronium. Although this remark does not resolve the orthopositronium decay puzzle, it allows for upper bounds to be set on neutrino magnetic moments.

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1. Positronium decay[1, 2] has provided for a number of years a somewhat uncomfortable crack in this otherwise most beautiful edifice which is QED. Indeed, the two most precise experimental rates for the  $C$ -odd triplet orthopositronium (o-PS) decay[3, 4],

$$\lambda_T^{\text{exp}} = 7.0514 \pm 0.0014 \mu\text{s}^{-1} \quad , \quad \lambda_T^{\text{exp}} = 7.0482 \pm 0.0016 \mu\text{s}^{-1} \quad , \quad (1)$$

are in flagrant conflict with the QED prediction[5, 6, 2],

$$\lambda_T^{\text{QED}}(\text{PS} \rightarrow 3\gamma) = 7.03831 \pm 0.00005 \mu\text{s}^{-1} \quad . \quad (2)$$

Note that the experimental results correspond, respectively, to a  $9.4\sigma$  and a  $6.2\sigma$  deviation from the theoretical expectation, and in relative terms, to the differences,

$$\frac{\lambda_T^{\text{exp}} - \lambda_T^{\text{QED}}}{\lambda_T^{\text{QED}}} = (1.86 \text{ or } 1.41) \cdot 10^{-3} \quad . \quad (3)$$

In contradistinction, the  $C$ -even singlet parapositronium (p-PS) decay rate, of which the most precise measurement gives[7],

$$\lambda_S^{\text{exp}} = 7990.0 \pm 1.7 \mu\text{s}^{-1} \quad , \quad (4)$$

compares better with the theoretical expectation[8, 6],

$$\lambda_S^{\text{QED}}(\text{PS} \rightarrow 2\gamma) = \frac{1}{2} \frac{\alpha^5 m_e c^2}{\hbar} \left[ 1 - \frac{\alpha}{\pi} \left( 5 - \frac{\pi^2}{4} \right) - 2\alpha^2 \ln \alpha + \dots \right] = 7989.5 \mu\text{s}^{-1} \quad . \quad (5)$$

Recently however, a new measurement of the o-PS decay rate has been published[9],

$$\lambda_T^{\text{exp}} = 7.0398 \pm 0.0025(\text{stat.}) \pm 0.0015(\text{syst.}) \mu\text{s}^{-1} \quad , \quad (6)$$

in good agreement with the value in (2), since it corresponds to a  $0.5\sigma$  deviation from the theoretical prediction and a relative difference with it of  $2.12 \cdot 10^{-4}$ . Clearly, an independent experimental confirmation of this beautiful result is desirable, before definitely concluding that the orthopositronium problem was indeed an experimental one.

Over the years, many possible explanations for the discrepancy in the o-PS decay rate have been suggested and analysed, both theoretically and experimentally[2]. Among these, one may mention mirror photons[10], decay modes involving weakly coupled (pseudo)scalar bosons or even so-called invisible decay modes. However, none of these possibilities can accomodate the complete o-PS discrepancy above. For example, (pseudo)scalar decay modes typically lead to the following upper limit on the o-PS branching ratio[2, 11],

$$\text{Br}(\text{o-PS} \rightarrow \gamma + X) \lesssim 10^{-4} - 10^{-6} \quad , \quad (7)$$

depending on the mass of the particle  $X$ , while the most stringent upper limit on the invisible branching ratio is[12, 2],

$$\text{Br}(\text{o-PS} \rightarrow \text{“invisible”}) < 2.8 \cdot 10^{-6} \quad . \quad (8)$$

Clearly, given the remark in (3), these upper bounds exclude the possibility of explaining the o-PS problem in terms of such decay modes only.

Confronted with this difficulty, theorists have returned back[13] to their calculations trying to establish the next correction<sup>1</sup> in  $(\alpha/\pi)^2$  to the perturbative expansions[5, 6, 2] used to determine the theoretical result in (2),

$$\lambda_T^{\text{QED}}(\text{PS} \rightarrow 3\gamma) = \frac{\alpha^6 m_e c^2}{\hbar} \frac{2(\pi^2 - 9)}{9\pi} \left[ 1 + (-10.282 \pm 0.003) \frac{\alpha}{\pi} + \frac{1}{3} \alpha^2 \ln \alpha + C \left( \frac{\alpha}{\pi} \right)^2 + \dots \right] . \quad (9)$$

However, in order to explain the observed discrepancy completely in terms of the next order calculation only, a coefficient  $C$  of the order of  $C \simeq 250 \pm 40$  is required[2]. Even though such a large coefficient is not to be excluded necessarily, such a value for the  $(\alpha/\pi)^2$  correction is difficult to contemplate. Incidentally, note that a value of  $C \simeq 40$  only is required by the recent new experimental result[9] given in (6).

Curiously enough, although the above discrepancy seems to affect only o-PS decay, none of the suggested mechanisms has tried to exploit this possible hint towards an explanation. One such instance is that of invisible decay modes involving neutrino-antineutrino pairs. Indeed, even when slightly massive, (anti)neutrinos emitted in positronium decay would be essentially (right)left-handed, since this is their chirality in the Standard Model (SM) whether there exists physics beyond it or not. Therefore, momentum and angular momentum conservation implies that neutrino decay of p-PS is strongly suppressed—by a factor  $(m_\nu/m_e)^2$ —or vanishing altogether in the Standard Model, whereas even in the SM o-PS does decay via neutrino pair emission, albeit with the typical small rate of a weak interaction process.

Obviously, in spite of the fact that this mechanism does indeed distinguish the decay of the two positronium hyperfine levels, it is expected to be far too small to explain the discrepancy for the o-PS rate, the relative weak interaction contribution having to be of the order  $(G_F m_e^2)^2/\alpha^3 \simeq 2.4 \cdot 10^{-17}$ ,  $G_F$  being Fermi's coupling constant. Nevertheless, the neutrino decay channel open essentially only to o-PS may be enhanced given additional couplings of the neutrinos. The possibility explicitly considered in this letter is that of non vanishing magnetic moments for Dirac neutrinos. Indeed,  $C$ -odd o-PS decay may then proceed through a single virtual photon which couples to the electron-positron pair and decays into a neutrino pair, while this additional decay channel is forbidden for the  $C$ -even p-PS state by charge conjugation invariance. Therefore, non vanishing neutrino magnetic moments would induce invisible decay modes of o-PS, but not of p-PS. In this way, given the measured o-PS decay rate, it is possible to establish upper bounds for neutrino magnetic moments. Such an analysis is the purpose of this letter<sup>2</sup>.

Present experimental upper limits on neutrino magnetic moments are<sup>3</sup> as follows[16],

$$\mu_{\nu_e} < 1.08 \cdot 10^{-9} \mu_B \quad , \quad \mu_{\nu_\mu} < 7.4 \cdot 10^{-10} \mu_B \quad , \quad \mu_{\nu_\tau} < 5.4 \cdot 10^{-7} \mu_B \quad , \quad (10)$$

$\mu_B$  being the Bohr magneton,  $\mu_B = e\hbar/2m_e$ . Let us also recall the value[18, 16] of the magnetic moment of a massive Dirac neutrino in the SM,

$$\mu_\nu = \frac{3eG_F m_e m_\nu}{8\pi^2 \sqrt{2}} = 3.2 \cdot 10^{-19} \left( \frac{m_\nu c^2}{1 \text{ eV}} \right) \mu_B \quad . \quad (11)$$

<sup>1</sup>It may be shown[14, 2] that  $5\gamma$ ,  $7\gamma$ , etc ..., decay modes of o-PS cannot explain the discrepancy either.

<sup>2</sup>To the authors' knowledge, the only other study of PS neutrino decay appears in Ref.[15]. However, that work addressed rather the radiative neutrino decay mode  $PS \rightarrow \nu \bar{\nu} \gamma$  only for massless neutrinos in the SM, and did not consider the possibility of non vanishing magnetic moments.

<sup>3</sup>Even though astrophysical or cosmological constraints lead to more stringent upper bounds[16, 17], such limits are model dependent and are not included here.

2. Let us first consider neutrino decay of positronium within the SM alone. For the purpose of applications beyond the SM however, neutrinos are taken to have a non vanishing Dirac mass already, but the possibility of flavour mixing will be ignored in this paper. The other assumptions entering the analysis are, on the one hand, that the couplings of these massive neutrinos to the  $W$  and  $Z$  are the usual ones as given by the SM, and on the other hand, that possible neutral Higgs contributions are not included since they are expected to be extremely small for light neutrinos. Relative to  $W$  and  $Z$  exchange diagrams, the neutral Higgs exchange amplitude is reduced by a factor  $m_e m_\nu / m_h^2$ , a very small ratio indeed. Finally, two further approximations are effected; on the one hand, the Ps binding energy is neglected compared to the  $e^-e^+$  total rest-mass energy  $2m_e c^2$ , namely the electron and positron are taken to annihilate at rest, and on the other hand, products of ratios of the electron or neutrino masses to the  $W$  and  $Z$  masses which appear in  $W$  and  $Z$  propagators are taken to be negligible as compared to unity.

These assumptions having been stated, only two amplitudes may contribute to the neutrino decay process. On the one hand, there is the  $Z$  exchange diagram which contributes for all neutrino flavours<sup>4</sup>  $\nu_\ell$  ( $\ell = e, \mu, \tau$ ). On the other hand, there is the charged current amplitude which contributes only to the electron flavour neutrino channel  $\nu_e$  via  $W$  exchange. A straightforward calculation then leads to the following contributions to the total decay rates. For the singlet state, one finds,

$$\lambda_S^{(ZZ)}(\nu_\ell) = \frac{\alpha^3}{16} \left( \frac{m_e^2 G_F}{\pi \sqrt{2}} \right)^2 \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_\ell}^2}{m_e^2}} \frac{m_{\nu_\ell}^2}{m_e^2} \quad , \quad \ell = e, \mu, \tau \quad , \quad (12)$$

$$\lambda_S^{(WW)}(\nu_e) = \frac{\alpha^3}{4} \left( \frac{m_e^2 G_F}{\pi \sqrt{2}} \right)^2 \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_e}^2}{m_e^2}} \frac{m_{\nu_e}^2}{m_e^2} \quad , \quad (13)$$

and,

$$\lambda_S^{(ZW)}(\nu_e) = \frac{\alpha^3}{4} \left( \frac{m_e^2 G_F}{\pi \sqrt{2}} \right)^2 \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_e}^2}{m_e^2}} \frac{m_{\nu_e}^2}{m_e^2} \quad , \quad (14)$$

where this last contribution follows from the interference of the  $W$  and  $Z$  exchange diagrams.

Similarly for the triplet state, one finds,

$$\lambda_T^{(ZZ)}(\nu_\ell) = \frac{\alpha^3}{12} \left( \frac{m_e^2 G_F}{\pi \sqrt{2}} \right)^2 \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_\ell}^2}{m_e^2}} \left( 1 - \frac{1}{4} \frac{m_{\nu_\ell}^2}{m_e^2} \right) (1 - 4 \sin^2 \theta_W)^2 \quad , \quad \ell = e, \mu, \tau \quad , \quad (15)$$

$$\lambda_T^{(WW)}(\nu_e) = \frac{\alpha^3}{3} \left( \frac{m_e^2 G_F}{\pi \sqrt{2}} \right)^2 \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_e}^2}{m_e^2}} \left( 1 - \frac{1}{4} \frac{m_{\nu_e}^2}{m_e^2} \right) \quad , \quad (16)$$

and for the interference contribution,

$$\lambda_T^{(ZW)}(\nu_\ell) = \frac{\alpha^3}{3} \left( \frac{m_e^2 G_F}{\pi \sqrt{2}} \right)^2 \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_\ell}^2}{m_e^2}} \left( 1 - \frac{1}{4} \frac{m_{\nu_\ell}^2}{m_e^2} \right) (1 - 4 \sin^2 \theta_W) \quad , \quad (17)$$

where  $\theta_W$  is the usual weak mixing angle for neutral currents.

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<sup>4</sup>For the sake of the analysis, three neutrino flavours whose mass is less than the electron mass are assumed to be involved in the decay process. Obviously, the argument would go through for an arbitrary number of neutrino flavours allowed by the decay kinematics.

A few remarks are in order. First, as was expected for reasons of the (right)left-handed chirality of (anti)neutrino couplings both to charged and to neutral currents, contributions to the singlet decay rate are all suppressed by a factor  $m_\nu^2/m_e^2$ . Second, even for the triplet state for which this chiral suppression is not effective,  $Z$  exchange contributions involve the factor  $(1 - 4 \sin^2 \theta_W = 0.0724)$  leading nevertheless to some suppression as well. And third, the relevant factor involving the weak coupling constant is  $(m_e^2 G_F / \pi \sqrt{2})^2 = 4.7 \cdot 10^{-25}$ , to be compared to the factors  $\alpha^2 = 5.33 \cdot 10^{-5}$  and  $(2(\pi^2 - 9)\alpha^3 / 9\pi) = 2.39 \cdot 10^{-8}$  relevant to the  $2\gamma$  and  $3\gamma$  decays of p-PS and o-PS, respectively. Consequently, neutrino decay of positronium in the SM is very much suppressed, beyond the reach of any experiment at present. For the sake of the illustration, in the case of a massless neutrino  $\nu_\ell$  ( $\ell \neq e$ ) one finds for example  $1/\lambda_T^{(ZZ)}(\nu_\ell, \ell \neq e) \simeq 5.14 \cdot 10^5$  years, to be compared to  $1/\lambda_T^{\text{QED}}(3\gamma) \simeq 142$  ns! Incidentally, note that factors of the form  $(1 - a m_\nu^2/m_e^2)$  with  $(a \simeq 1)$ , do not differ significantly from unity for neutrino masses less than say, a fifth of the electron mass.

3. Let us now extend the above analysis to include the possibility of neutrino magnetic moments. As is well known[19], this requires massive neutrinos of the Dirac type, hence necessarily new physics beyond that of the Standard Model. The effective magnetic moment coupling of Dirac neutrinos is of the form,

$$\overline{\psi}_\nu \frac{1}{2} \mu_\nu \sigma^{\alpha\beta} F_{\alpha\beta} \psi_\nu \quad , \quad (18)$$

where  $F_{\alpha\beta}$  is the usual electromagnetic field strength  $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ , and  $\mu_\nu$  the neutrino (anomalous) magnetic moment[20]. Given this coupling, positronium decay then proceeds via one more amplitude in addition to those above, namely through the single photon annihilation channel  $e^- e^+ \rightarrow \gamma \rightarrow \nu_\ell \overline{\nu}_\ell$ . In the decay rate, this photon amplitude also interferes with the charged and neutral current amplitudes of the Standard Model.

For the singlet state, one then finds the identically vanishing contributions,

$$\lambda_S^{(\gamma\gamma)}(\nu_\ell) = 0 \quad , \quad \ell = e, \mu, \tau \quad , \quad (19)$$

$$\lambda_S^{(\gamma Z)}(\nu_\ell) = 0 \quad , \quad \ell = e, \mu, \tau \quad , \quad (20)$$

$$\lambda_S^{(\gamma W)}(\nu_e) = 0 \quad , \quad (21)$$

as is indeed required by charge conjugation invariance of electromagnetic couplings. On the other hand for the triplet state, one obtains,

$$\lambda_T^{(\gamma\gamma)}(\nu_\ell) = \frac{\alpha^3}{12} \left( \alpha \frac{\mu_{\nu_\ell}}{\mu_B} \right)^2 \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_\ell}^2}{m_e^2}} \left( 1 + 2 \frac{m_{\nu_\ell}^2}{m_e^2} \right) \quad , \quad \ell = e, \mu, \tau \quad , \quad (22)$$

$$\lambda_T^{(\gamma Z)}(\nu_\ell) = \frac{\alpha^3}{4} \left( \frac{m_e^2 G_F}{\pi \sqrt{2}} \right) \left( \alpha \frac{\mu_{\nu_\ell}}{\mu_B} \right) \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_\ell}^2}{m_e^2}} \frac{m_{\nu_\ell}}{m_e} (1 - 4 \sin^2 \theta_W) \quad , \quad \ell = e, \mu, \tau \quad , \quad (23)$$

and

$$\lambda_T^{(\gamma W)}(\nu_e) = \frac{\alpha^3}{2} \left( \frac{m_e^2 G_F}{\pi \sqrt{2}} \right) \left( \alpha \frac{\mu_{\nu_e}}{\mu_B} \right) \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_e}^2}{m_e^2}} \frac{m_{\nu_e}}{m_e} \quad . \quad (24)$$

In these expressions,  $\mu_B = e\hbar/2m_e$  is the Bohr magneton. Note that the  $(\gamma Z)$  and  $(\gamma W)$  interference contributions involve directly the ratio  $m_\nu/m_e$ , in contradistinction to the pure  $(\gamma\gamma)$  contribution. The reason for this result is that the magnetic neutrino coupling implies

a spin flip whereas the  $W$  and  $Z$  (anti)neutrino couplings are purely (right)left-handed; angular momentum conservation thus requires one insertion of the neutrino mass vertex operator  $m_\nu \bar{\psi}_\nu \psi_\nu$ . Hence, given small neutrino masses as compared to the electron mass, it is essentially the  $(\gamma\gamma)$  contribution which dominates over all other magnetic moment contributions, provided the factor  $\alpha\mu_\nu/\mu_B$  is of the order of the effective weak coupling ( $m_e^2 G_F/\pi\sqrt{2}$ ) or larger.

4. Given the above results, the total decay rates for both singlet and triplet states into neutrino flavours are obtained as, respectively,

$$\lambda_S(p - \text{PS} \rightarrow \nu\bar{\nu}) = \sum_{\ell=e,\mu,\tau} \lambda_S^{(ZZ)}(\nu_\ell) + \left[ \lambda_S^{(WW)}(\nu_e) + \lambda_S^{(ZW)}(\nu_e) \right] \quad , \quad (25)$$

and,

$$\begin{aligned} \lambda_T(o - \text{PS} \rightarrow \nu\bar{\nu}) &= \sum_{\ell=e,\mu,\tau} \left[ \lambda_T^{(\gamma\gamma)}(\nu_\ell) + \lambda_T^{(\gamma Z)}(\nu_\ell) + \lambda_T^{(ZZ)}(\nu_\ell) \right] + \\ &+ \left[ \lambda_T^{(\gamma W)}(\nu_e) + \lambda_T^{(WW)}(\nu_e) + \lambda_T^{(ZW)}(\nu_e) \right] \quad . \end{aligned} \quad (26)$$

In particular, in the limit of vanishing neutrino masses, these expressions reduce to,

$$\lambda_S(p - \text{PS} \rightarrow \nu\bar{\nu}) = 0 \quad , \quad (27)$$

and,

$$\begin{aligned} \lambda_T(o - \text{PS} \rightarrow \nu\bar{\nu}) &= \frac{\alpha^3}{12} \frac{m_e c^2}{\hbar} \sum_{\ell=e,\mu,\tau} \left( \alpha \frac{\mu_{\nu_\ell}}{\mu_B} \right)^2 + \\ &+ \frac{\alpha^3}{3} \left( \frac{m_e^2 G_F}{\pi\sqrt{2}} \right)^2 \frac{m_e c^2}{\hbar} \left[ 1 + (1 - 4 \sin^2 \theta_W) + \frac{1}{4} N_\nu (1 - 4 \sin^2 \theta_W)^2 \right] \quad , \end{aligned} \quad (28)$$

where  $N_\nu = 3$  is the number of light neutrinos. Note that these two expressions confirm the announced result, namely the fact that in practice neutrino disintegration of positronium is a decay channel open essentially only to the triplet hyperfine state.

The SM contribution from  $W$  and  $Z$  exchange relative to the QED  $3\gamma$  decay rate being  $7.2 \cdot 10^{-17}$ , let us first assume that the neutrino magnetic moment contributions to  $\lambda_T(o - \text{PS} \rightarrow \nu\bar{\nu})$  are dominant, namely let us restrict the discussion now to the first term only in (28). Compared to the QED decay rate  $\lambda_T^{\text{QED}}(\text{PS} \rightarrow 3\gamma)$  in (9), the quantity relevant for the confrontation with experimental results is thus,

$$\frac{\frac{\alpha^3}{12} \frac{m_e c^2}{\hbar} \sum_{\ell=e,\mu,\tau} \left( \alpha \frac{\mu_{\nu_\ell}}{\mu_B} \right)^2}{\lambda_T^{\text{QED}}(\text{PS} \rightarrow 3\gamma)} = 3.6 \cdot 10^6 \sum_{\ell=e,\mu,\tau} \left( \alpha \frac{\mu_{\nu_\ell}}{\mu_B} \right)^2 \quad . \quad (29)$$

Taken at face value, if this ratio were to explain completely the discrepancy expressed in relative terms in (3), one would require the following limit,

$$\sqrt{\sum_{\ell=e,\mu,\tau} \mu_{\nu_\ell}^2} \lesssim 3 \cdot 10^{-3} \mu_B \quad . \quad (30)$$

On the other hand, if the corresponding limit stemming from the more recent measurement in (6) is used in the same manner, one derives,

$$\sqrt{\sum_{\ell=e,\mu,\tau} \mu_{\nu_\ell}^2} \lesssim 10^{-3} \mu_B \quad . \quad (31)$$

However, since the upper limit on the invisible branching ratio in (8) already excludes such possibilities, it is more sensible to use the constraint on this mode to set the upper bound,

$$\sqrt{\sum_{\ell=e,\mu,\tau} \mu_{\nu_\ell}^2} \lesssim 1.2 \cdot 10^{-4} \mu_B \quad . \quad (32)$$

Clearly, these numbers are not competitive with the experimental limits on neutrino magnetic moments in (10). Even for the least stringent upper bound which applies in the case of the neutrino  $\nu_\tau$ ,  $\mu_{\nu_\tau} < 5.4 \cdot 10^{-7} \mu_B$ , the corresponding branching ratio is already,

$$Br(o - PS \rightarrow \nu_\tau \bar{\nu}_\tau) < 5.5 \cdot 10^{-11} \quad , \quad (33)$$

namely much less than present experimental upper limits on branching ratios for invisible decay modes or for the relative experimental deviations from the QED prediction.

As a matter of fact, it is instructive to consider the situation when the magnetic moment contribution to the neutrino branching ratio is comparable to that of the charged and neutral currents, namely when  $\alpha \mu_\nu / \mu_B = m_e^2 G_F / \pi \sqrt{2}$ . Indeed, the corresponding magnetic moment value,

$$\mu_\nu = 9.39 \cdot 10^{-11} \mu_B \quad , \quad (34)$$

is of the same order of magnitude as the best experimental upper limit in (10) established for the muon neutrino. *A posteriori*, this is not surprising, since these experimental limits are determined from neutrino scattering experiments designed to be sensitive to processes whose strength is typical of weak interactions. Hence, present limits on neutrino magnetic moments necessarily correspond to values which render their contributions comparable to those of the ordinary charged and neutral electroweak currents. In the present case, the magnetic moment value in (34) for a single neutrino contributes the invisible branching ratio the quantity  $1.7 \cdot 10^{-18}$ , which is indeed close to the neutrino branching ratio in the SM of  $7.2 \cdot 10^{-18}$ , namely the contribution of the second term in (28).

In fact, using the upper limits in (10) as values for the neutrino magnetic moments together with the result in (28), one obtains the following total neutrino branching ratio,

$$Br(o - PS \rightarrow \nu \bar{\nu}) = 5.6 \cdot 10^{-11} \quad , \quad (35)$$

which is thus dominated by the magnetic moment contribution of the  $\nu_\tau$  neutrino.

However, the value in (35) is larger than upper bounds recently established[21] for the branching ratio of “exotic” positronium decays on the basis of primordial nucleosynthesis. In fact, since the electroweak charged and neutral current contributions relevant to the thermal equilibrium of three light neutrinos are already included[22] in the standard cosmological model, the “exotic” contributions to be considered here are solely those stemming from the neutrino magnetic moments. Moreover, the associated couplings (18) being dimension five operators, it is actually the upper bound of  $2 \cdot 10^{-17}$  on the branching ratio, associated to case B) of Ref.[21], which is relevant in our case and which thus applies only to the total contribution of neutrino magnetic moments. Correspondingly, this upper limit implies,

$$\sqrt{\sum_{\ell=e,\mu,\tau} \mu_{\nu_\ell}^2} \lesssim 3.2 \cdot 10^{-10} \mu_B \quad . \quad (36)$$

Incidentally, note that this upper bound value is also typical of astrophysical constraints on neutrino magnetic moments[16].

5. In conclusion, even though neutrino decay of positronium in the Standard Model and beyond it does indeed contribute to invisible decay modes of the triplet orthopositronium state, but practically not to the decay of the singlet parapositronium state, present experimental limits on neutrino magnetic moments imply that such processes cannot provide even for a partial contribution towards a resolution of the possible orthopositronium lifetime puzzle[2], which however, may have been resolved by a recent new measurement[9]. Moreover, these limits also establish that the observation of such decay modes is far beyond present experimental capabilities. Nevertheless, when considered together with recent arguments[21] based on primordial nucleosynthesis, the orthopositronium neutrino decay mode implies upper bounds on neutrino magnetic moments which are more stringent than present experimental limits.

Although the analysis assumes massive Dirac neutrinos without flavour mixing, it should be clear that similar conclusions would apply more generally for transition magnetic moments as well, whether the massive neutrinos are Dirac or Majorana spinors with flavour mixing.

Note also that an analogous discussion could be developed were neutrinos to carry electric charges[23]  $eQ_\nu$ . Under such circumstances, the associated couplings would imply contributions to positronium decay such as those above in which the ratio  $\alpha\mu_\nu/\mu_B$  is replaced by the factor  $\alpha Q_\nu$ , thereby leading to similar conclusions. For example in the case of the triplet decay rate<sup>5</sup>, the modulus squared photon amplitude leads to,

$$\lambda_T^{(\gamma\gamma)}(\nu_\ell; Q_{\nu_\ell}) = \frac{1}{6} \alpha^3 (\alpha Q_{\nu_\ell})^2 \frac{m_e c^2}{\hbar} \sqrt{1 - \frac{m_{\nu_\ell}^2}{m_e^2}} \left( 1 + \frac{1}{2} \frac{m_{\nu_\ell}^2}{m_e^2} \right) \quad , \quad \ell = e, \mu, \tau \quad . \quad (37)$$

Therefore, the same types of upper bounds as those derived above for the ratios  $\mu_\nu/\mu_B$  would apply to the neutrino electric charges  $Q_\nu$ . However, these constraints are not competitive with existing limits[23]. In this respect, it is of interest to remark that based on the anisotropy of the microwave background, an upper bound of  $|Q_\nu| < 4.8 \cdot 10^{-34}$  was recently obtained[24], which is thus many orders of magnitude more stringent than any existing limit on neutrino magnetic moments.

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<sup>5</sup>Here again, contributions to the singlet rate vanish by charge conjugation invariance.



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